Intuitionistic Fuzzy P-Ideals of BCI-Algebras with Interval Valued Membership& Non Membership Functions

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Abstract: *The purpose of this paper is to define the notion of an interval valued Intuitionistic Fuzzy P-ideal (briefly, an i-v IF P-ideal) of a BCI – algebra. Necessary and sufficient conditions for an i-v Intuitionistic Fuzzy P-idealare stated. Cartesian product of i-v Fuzzy ideals are discussed.*

Keywords: BCI-algebra, P-ideal, i-v intuitionistic fuzzy P-ideals

1. Introduction:

The notion of BCK-algebras was proposed by Imai and Iseki in 1996. In the same year, Iseki [6] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. Since then numerous mathematical papers have been written investigating the algebraic properties of the BCK/BCI-algebras and their relationship with other universal structures including lattices and Boolean algebras. Fuzzy sets were initiated by Zadeh [10]. In [9], Zadeh made an extension of the concept of a Fuzzy set by an interval-valued fuzzy set. This interval-valued fuzzy set is referred to as an i-v fuzzy set. InZadeh also constructed a method of approximate inference using his i-v fuzzy sets. In Birwa's defined interval valued fuzzy subgroups of Rosenfeld`s nature, and investigated some elementary properties. The idea of "intuitionistic fuzzy set" was first published by Atanassov as a generalization of notion of fuzzy sets. After that many researchers considers the Fuzzifications of ideal and sub algebras in BCK/BCI-algebras. In this paper, using the notion of interval valued fuzzy set, we introduce the concept of an interval-valued intuitionistic fuzzy BCI-algebra of a BCI-algebra, and study some of their properties. Using an i-v level set of i-v intuitionistic fuzzy set, we state a characterization of an intuitionistic fuzzy P-ideal of BCI-algebra. We prove that every intuitionistic fuzzy P-ideal of a BCI-algebra X can be realized as an i-v level P-ideal of an i-v intuitionistic fuzzyP-ideal of X. in connection with the notion of homomorphism, we study how the images and inverse images of i-v intuitionistic fuzzy P-ideal become i-v intuitionistic fuzzy P-ideal.

2. Preliminaries:

Let us recall that an algebra (X,*,0) of type (2,0) is called a BCI-algebra if it satisfies the following conditions:1.((x*y)*(x*z))*(z*y)=0,2.(x*(x*y))*y=0,3.x*x=0,4.x*y=0 and y*x=0 imply x=y,for all x,y,z ϵ X.
In a BCI-algebra, we can define a partial ordering"≤" by x≤y if and only if x*y=0.in a BCI-algebra X, the set M={xϵX/0*x=0} is a sub algebra and is called the BCK-part of X. A BCI-algebra X is called proper if X-M≠ϕ. otherwise it is improper. Moreover, in a BCI-algebra the following conditions hold:
1. (x*y)*z=(x*z)*y, 2.x*0=0, 3. x ≤y imply x*z ≤y*z and z*y ≤z*x, 4. 0*(x*y) = (0*x)*(0*y),
5. 0*(x*y) = (0*x)*(0*y), 6. 0*(0*(x*y)) =0*(y*x), 7. (x *z)*(y*z) ≤x*y
An intuitionistic fuzzy set A in a non-empty set X is an object having the form A= {<x,μ_A(x),υ_A(x)>/xϵX},Where the functions μ_A : X→[0,1] and υ_A: X→[0,1] denote the degree of the membership and the degree of non membershipof each element xϵ X to the set A respectively, and 0≤ μ_A(x) +υ_A(x) ≤ 1 for all xϵ X.Such defined objects are studied by many authors and have many interesting applications not only in the mathematics. For the sake of simplicity, we shall use the symbol A=[μ_A, υ_A] for the intuitionistic fuzzy set A={[μ_A(x),υ_A(x)]/ xϵX}.

Definition 2.1:A non empty subset I of X is called an ideal of X if it satisfies:1. 0ϵI,2.x*yϵI and yϵI ⇒ xϵI.

Definition 2.2:A fuzzy subset μ of a BCI-algebra X is called afuzzy ideal of X if it satisfies:
1.μ(0)≥μ(x), 2. μ(x)≥min {μ(x*y),μ(y)},for all x,yϵX.

Definition 2.3: A non empty subset I of X is called anP- ideal of X if it satisfies:
1. 0ϵI. 2. (x*z)*(y*z)ϵI and yϵI imply x*zϵI.Putting z=0 in(2) then we see that every P- ideal is an ideal.

Definition 2.4:A fuzzy set μ in a BCI-algebra X is called an fuzzyP- ideal of X if
1.μ(0)≥μ(x), 2. μ(x)≥min {μ ((x*z)*(y*z)),μ(y)}.

Definition 2.5: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A \cap B$ membership function $\mu_{A\cap B}$ is defined by $\mu_{A\cap B}(\mathrm{x}) = \min\{\mu_A(\mathrm{x}), \mu_B(\mathrm{x})\}, x \in X$.

Definition 2.6: Let A and B be two fuzzy ideal of BCI algebra X. The fuzzy set $A \cup B$ with membership function $\mu_{A\cup B}$ is defined by $\mu_{A\cup B}(\mathrm{x}) = \max\{\mu_A(\mathrm{x}), \mu_B(\mathrm{x})\}, \forall x \in X$.

Definition 2.7: Let A and B be two fuzzy ideal of BCI algebra X with membership functionand respectively. A is contained in B if $\mu_A(x) \le \mu_B(x), \ \forall \ x \in X$

Definition 2.8: Let A be a fuzzy ideal of BCI algebra X.The fuzzy set A^m with membership function μ_{A^m} is defined by $\mu_{A^m}(x) \le (\mu_A(x))^m, \forall x \in X$

Definition 2.9: An IFS A=< X, μ_A,υ_A> in a BCI-algebra X is called an intuitionisticfuzzy ideal of X if it satisfies:(F1) $\mu_A(0) \ge \mu_A(x)$ &$\upsilon_A(0) \ge \upsilon_A(x)$,(F2) $\mu_A(x) \ge \min\{\mu_A(x*y), \mu_A(y)\}$,

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(F3) $\upsilon_A(x) \le \max\{\upsilon A(x*y), \upsilon_A(y)\}$, for all x,y$\epsilon$X

Definition 2.10: An intuitionistic fuzzy set A=< μ_A, υ_A> of a BCI-algebra X is called an intuitionistic fuzzy P-ideal if it satisfies (F1) and(F4) $\mu_A(x) \ge \min\{\mu_A((x*z)*(y*z)), \mu_A(y)\}$,(F5) $\upsilon_A(y*x) \le \max\{\upsilon_A((x*z)*(y*z)), \upsilon_A(y)\}$, for all x,y,z ϵ X.

An interval-valued intuitionistic fuzzy set A defined on X is given byA=$\{(x,[\mu_A^L(x)\mu_A^U(x)],[\upsilon_A^L(x)\upsilon_A^U(x)])\},\forall x\epsilon X$ where μ_A^L,μ_A^U are two membership functions and υ_A^L,υ_A^U are two non-membership functions X such that $\mu_A^L \le \mu_A^U$&$\upsilon_A^L \ge \upsilon_A^U$,$\forall$ xϵX. Let $\bar{\mu}_A(x)=[\mu_A^L,\mu_A^U]$&$\overline{\upsilon_A}(x)=[\upsilon_A^L,\upsilon_A^U]$,$\forall$ xϵX and let D[0,1]denote the family of all closed subintervals of [0,1].If $\mu_A^L(x)=\mu_A^U(x)$=c,0≤c≤1 and if $\upsilon_A^L(x)=\upsilon_A^U(x)$=k, 0≤k≤1,then we have $\bar{\mu}_A(x)$=[c,c]&$\bar{\upsilon}_A(x)$=[k,k] which we also assume, for the sake of convenience, to belong to D[0,1]. thus $\bar{\mu}_A(x)$&$\bar{\upsilon}_A(x)\epsilon$[0,1],$\forall$ xϵX,and therefore the i-v IFS a is given by A=$[(x,\bar{\mu}_A(x),\bar{\upsilon}_A(x))\}$,$\forall$ xϵX,where $\bar{\mu}_A(x)$:X→D[0,1]. Now let us define what is known as refined minimum, refined maximum of two elements in D[0,1].we also define the symbols" ≤","≥" and "=" in the case of two elements in D[0,1]. Consider two elements D_1:$[a_1,b_1]$and D_2:$[a_2,b_2]\epsilon$D[0,1]. Then rmin(D_1,D_2)=[min{a_1,a_2},min{b_1,b_2}],rmax(D_1,D_2)=[max{a_1,a_2},max{b_1,b_2}]$D_1 \ge D_2 \Leftrightarrow$ $a_1 \ge a_2, b_1 \ge b_2$;$D_1 \le D_2 \Leftrightarrow$ $a_1 \le a_2, b_1 \le b_2$ and $D_1=D_2$.

3.Interval-valued Intuitionistic fuzzy P-ideals of BCI-algebras

Definition 3.1: An interval-valued intuitionistic fuzzy set A in BCI-algebra X is called an interval-valued intuitionistic fuzzy P-ideal of X if it satisfies (FI_1)$\bar{\mu}_A(0) \ge \bar{\mu}_A(x)$,$\bar{\upsilon}_A(0) \le \bar{\upsilon}_A(x)$,
(FI_2)$\bar{\mu}_A(x) \ge$r min $\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\}$, (FI_3)$\bar{\upsilon}_A(x) \le$ r max $\{\bar{\upsilon}_A((x*z)*(y*z)),\bar{\upsilon}_A(y)\}$.

Theorem 3.2 Let A be an i-v intuitionistic fuzzyP-ideal of X. if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}\bar{\mu}_A(x_n)$=[1,1], $\lim_{n\to\infty}\bar{\nu}_A(x_n)$=[0,0] then $\bar{\mu}_A(0)$=[1,1] and$\bar{\upsilon}_A(0)$=[0,0].

Proof: Since $\bar{\mu}_A(0) \ge \bar{\mu}_A(x)$and $\bar{\upsilon}_A(0) \le \bar{\upsilon}_A(x)$ for all xϵX, we have $\bar{\mu}_A(0) \ge \bar{\mu}_A(x_n)$ and $\bar{\upsilon}_A(0) \le \bar{\upsilon}_A(x_n)$, for every positive integer n. note that$[\mu_A^L,\mu_A^U] \ge \bar{\mu}_A(0)$.[1,1] $\ge \bar{\mu}_A(x) \ge \bar{\mu}_A(0) \ge \lim_{n\to\infty}\bar{\mu}_A(x_n)$=[1,1]. $[\lambda_A^L,\lambda_A^U] \le \bar{\lambda}_A(0)$.[0,0] $\le \bar{\upsilon}_A(x) \le \bar{\upsilon}_A(0) \le \lim_{n\to\infty}\bar{\nu}_A(x_n)$=[0,0].Hence $\bar{\mu}_A(0)$=[1,1] and$\bar{\upsilon}_A(0)$=[0,0].

Lemma3.3: An i-v intuitionistic fuzzy set A= $[\langle\mu_A^L,\mu_A^U\rangle,\langle\upsilon_A^L,\upsilon_A^U\rangle]$ in X is an i-v intuitionistic fuzzy P-ideal of X if and only if $\langle\mu_A^L,\mu_A^U\rangle$and $\langle\upsilon_A^L,\upsilon_A^U\rangle$ are intuitionistic fuzzy ideals of X.

Proof: Since $\mu_A^L(0) \ge \mu_A^L(x)$; $\mu_A^U(0) \ge \mu_A^U(x)$;$\upsilon_A^L(0) \le \upsilon_A^L(x)$and $\upsilon_A^U(0) \le \upsilon_A^U(x)$,
Therefore$\bar{\mu}_A(0) \ge \bar{\mu}_A(x)$,$\bar{\upsilon}_A(0) \le \bar{\upsilon}_A(x)$.
Suppose that $\langle\mu_A^L,\mu_A^U\rangle$ and $\langle\upsilon_A^L,\upsilon_A^U\rangle$are intuitionistic fuzzy ideal of X. let x,yϵX, then
$\bar{\mu}_A(x)=[\mu_A^L(x),\mu_A^U(x)] \ge [\min\{\mu_A^L(x*y),\mu_A^L(y)\},\min\{\mu_A^U(x*y),\mu_A^U(y)\}]$
=r min $\{[\mu_A^L(x*y),\mu_A^U(x*y)],[\mu_A^L(y),\mu_A^U(y)]\}$
= r min $\{\bar{\mu}_A(x*y),\bar{\mu}_A(y)\}$ and
$\bar{\upsilon}_A(x)=[\upsilon_A^L(x),\upsilon_A^U(x)] \le [\max\{\upsilon_A^L(x*y),\upsilon_A^L(y)\},\max\{\upsilon_A^U(x*y),\upsilon_A^U(y)\}]$
=r max $\{[\upsilon_A^L(x*y),\upsilon(x*y)],[\upsilon_A^L(y),\upsilon(y)]\}$
= r max $\{\bar{\upsilon}_A(x*y),\bar{\upsilon}_A(y)\}$.
Hence A is an i-v intuitionistic fuzzy ideal of X.
Conversely,
Assume that A is an i-v intuitionistic fuzzy ideal of X. for any x,yϵX,we have
$[\mu_A^L(x),\mu_A^U(x)]=\bar{\mu}A(x) \ge$ r min$\{[\bar{\mu}_A(x*y),\bar{\mu}_A(y)]\}$
=r min $\{[\mu_A^L(x*y),\mu_A^U(x*y)],[\mu_A^L(y),\mu_A^U(y)]\}$
= [min $\{\mu_A^L(x*y),\mu_A^L(y)\}$,min$\{\mu_A^U(x*y),\mu_A^U(y)\}$]
And $[\upsilon_A^L(x),\upsilon_A^U(x)]=\bar{\upsilon}_A(x) \le$ r max$\{\bar{\upsilon}_A(x*y),\bar{\upsilon}_A(y)\}$
=r max $\{[\upsilon_A^L(x*y),\upsilon_A^U(x*y)],[\upsilon_A^L(y),\upsilon_A^U(y)]\}$
= [max $\{\upsilon_A^L(x*y),\upsilon_A^L(y)\}$,min$\{\upsilon_A^U(x*y),\upsilon_A^U(y)\}$]

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It follows thatμ_A^L(x) ≥ min {μ_A^L(x*y),μ_A^L(y)},υ_A^L(x) ≤ max{υ_A^L(x*y),υ_A^L(y)}
Andμ_A^U(x) ≥ min {μ_A^U(x*y),μ_A^U(y)}, υ_A^U(x) ≤max {υ_A^U(x*y),υ_A^U(y)}
Hence ⟨μ_A^L, μ_A^U⟩and ⟨$\upsilon_A^L, \upsilon_A^U$⟩ are intuitionistic fuzzy ideals of X.

Theorem 3.4. Every i-v intuitionistic fuzzy P-ideal of a BCI-algebraX is an i-v intuitionistic fuzzy ideal.

Proof: Let A=[⟨μ_A^L, μ_A^U⟩, ⟨$\upsilon_A^L, \upsilon_A^U$⟩] be an i-v intuitionistic fuzzy P-ideal of X, where⟨μ_A^L, μ_A^U⟩and ⟨$\upsilon_A^L, \upsilon_A^U$⟩are intuitionistic fuzzy P-ideal of X. thus ⟨μ_A^L, μ_A^U⟩and ⟨$\upsilon_A^L, \upsilon_A^U$⟩are intuitionistic fuzzy P-ideals of X. hence by lemma 3.3, A is i-v intuitionistic fuzzy ideal of X**.**

Definition 3.5: An i-v intuitionistic fussy set A in X is called an interval-valued intuitionistic fuzzy BCI-sub algebraof Xif $\bar{\mu}_A$(x*y)≥r min { $\bar{\mu}_A$(x), $\bar{\mu}_A$(y)} and $\bar{\upsilon}_A$ (x*y)≤ {$\bar{\upsilon}_A$(x),$\bar{\upsilon}_B$(y)}, for all x,yϵX.

Theorem 3.6: Every i-v intuitionisticfuzzy P-ideal of a BCI-algebra X is i-v intuitionistic fuzzy sub algebra of X.**Proof:**Let A=[⟨μ_A^L, μ_A^U⟩, ⟨$\upsilon_A^L, \upsilon_A^U$⟩]be an i-v intuitionistic fuzzy P-ideal of X, where ⟨μ_A^L, μ_A^U⟩, and ⟨$\upsilon_A^L, \upsilon_A^U$⟩ are intuitionistic fuzzy P-ideal of BCI-algebra X. thus ⟨μ_A^L, μ_A^U⟩, and ⟨$\upsilon_A^L, \upsilon_A^U$⟩are intuitionistic fuzzy subalgebra of X. Hence, A is i-v intuitionistic fuzzysub algebra of X.

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4. Cartesian product of i-v intuitionistic fuzzy P-ideals

Definition 4.1 An intuitionistic fuzzy relation A on any set a is a intuitionistic fuzzy subset A with a membership function Ω_A: X×X→ [0, 1] and non membership function Ψ_A: X×X→ [0, 1].

Lemma 4.2 Let $\bar{\mu}_A$and $\bar{\mu}_B$ be two membership functions and $\bar{\upsilon}_A$and$\bar{\upsilon}_B$ be two non membership functions of each x ϵX to the i-v subsets A and B, respectively. Then $\mu_A \times \mu_B$ is membership function and $\upsilon_A \times \upsilon_B$ is non membership function of each element(x,y)ϵX×X to the set A×B and defined by ($\bar{\mu}_A \times \bar{\mu}_B$)(x,y)=r min {$\bar{\mu}_A$(x), $\bar{\mu}_B$(y)} and ($\bar{\upsilon}_A \times \bar{\upsilon}_B$)(x,y)=r max {$\bar{\upsilon}_A$(x),$\bar{\upsilon}_B$ (y)}.

Definition 4.3 Let A= [⟨μ_A^L, μ_A^U⟩, ⟨$\upsilon_A^L, \upsilon_A^U$⟩]and B=[⟨$\mu_B^L, \mu_B^U$⟩, ⟨$\upsilon_B^L, \upsilon_B^U$⟩] be two i-v intuitionistic fuzzy subsets in a set X. The Cartesian product of A×B is defined byA×B= {((x,y), $\bar{\mu}_A \times \bar{\mu}_B$, $\bar{\upsilon}_A \times \bar{\upsilon}_B$);∀x,y$\epsilon$X×X}Where A×B: X×X→D[0,1].

Theorem 4.4. Let A=[⟨μ_A^L, μ_A^U⟩, ⟨$\upsilon_A^L, \upsilon_A^U$⟩]and B=[⟨$\mu_B^L, \mu_B^U$⟩, ⟨$\upsilon_B^L, \upsilon_B^U$⟩] be two i-v intuitionistic fuzzy subsets in a set X,then A×B is an i-v intuitionistic fuzzy P-ideal of X×X.

Proof: Let(x,y) ϵX×X, then by definition

$$(\bar{\mu}_A\times\bar{\mu}_B)(0,0)=r\min\{\bar{\mu}_A(0),\bar{\mu}_B(0)=r\min\{[\mu_A^L(0),\mu_A^U(0)],[\mu_B^L(0),\mu_B^U(0)]\}$$
$$=[\min\{\mu_A^L(0),\mu_B^L(0)\},\min\{\mu_A^U(0),\mu_B^U(0)\}]$$
$$\geq[\min\{\mu_A^L(x),\mu_B^L(y)\},\min\{\mu_A^U(x),\mu_B^U(y)\}]$$
$$=r\min\{[\mu_A^L(x),\mu_A^U(x)],[\mu_B^L(y),\mu_B^U(y)]\}$$
$$=r\min\{\bar{\mu}_A(x),\bar{\mu}_B(y)\}=(\bar{\mu}_A\times\bar{\mu}_B)(x,y)$$

And
$$(\bar{\upsilon}_A\times\bar{\upsilon}_B)(0,0)=r\max\{\bar{\upsilon}_A(0),\bar{\upsilon}_B(0)$$
$$=r\max\{[\upsilon_A^L(0),\upsilon_A^U(0)],[\upsilon_B^L(0),\upsilon_B^U(0)]\}$$
$$=[\max\{\upsilon_A^L(0),\upsilon_B^L(0)\},\max\{\upsilon_A^U(0),\upsilon_B^U(0)\}]$$
$$\leq[\max\{\upsilon_A^L(x),\upsilon_B^L(y)\},\max\{\upsilon_A^U(x),\upsilon_B^U(y)\}]$$
$$=r\max\{[\upsilon_A^L(x),\upsilon_A^U(x)],[\upsilon_B^L(y),\upsilon_B^U(y)]\}$$
$$=r\max\{\bar{\upsilon}_A(x),\bar{\upsilon}_B(y)=(\bar{\upsilon}_A\times\bar{\upsilon}_B)(x,y)$$

Therefore (FI_2) holds.Now, for all x,y,zϵX, we have

$$(\bar{\mu}_A\times\bar{\mu}_B)((x,x^{'}))=r\min\{\mu_A(x),\mu_B(x^{'})\}$$
$$\geq r\min\{r\min\{\bar{\mu}_A((x*z)*(y*z)),\bar{\mu}_A(y)\},r\min\{\bar{\mu}_A((x^1*z^1)*(y^1*z^1)),\bar{\mu}_A(y^1)\}\}$$
$$=r\min\{\{\min\{\mu^L{}_A((x*z)*(y*z)),\mu^L{}_A(y)\},\ \min\{\mu^U{}_A((x*z)*(y*z)),\mu^U{}_A(y)\}\},$$
$$\{\min\{\mu^L{}_B((x^1*z^1)*(y^1*z^1)),\mu^L{}_B(y^1)\},\min\{\mu^U{}_B((x^1*z^1)*(y^1*z^1)),\mu^U{}_B(y^1)\}\}$$
$$=\{\min\{\min\{\mu^L{}_A((x*z)*(y*z)),\mu^L{}_B((x^1*z^1)*(y^1*z^1))\},\min\{\mu^L{}_A(y),\ \mu^L{}_B(y^1)\}\},$$
$$\min\{\min\{\mu^U{}_A((x*z)*(y*z)),\mu^U{}_B((x^1*z^1)*(y^1*z^1))\},\min\{\mu^U{}_A(y),\mu^U{}_B(y^1)\}\}\}$$
$$=r\min\{(\bar{\mu}_A\times\bar{\mu}_B)(((x*z)*(y*z)),\ ((x^1*z^1)*(y^1*z^1))),(\bar{\mu}_A\times\bar{\mu}_B)(y,y^{'})\}$$

Also,
$$(\bar{\upsilon}_A\times\bar{\upsilon}_B)((x,x^{'}))=r\max\{\upsilon_A(x),\upsilon_B(x^{'})\}$$
$$\leq r\max\{r\max\{\bar{\nu}_A((x*z)*(y*z)),\bar{\nu}_A(y)\},r\max\{\bar{\nu}_A((x^1*z^1)*(y^1*z^1)),\bar{\nu}_A(y^1)\}\}$$
$$=r\max\{\{\max\{\nu^L{}_A((x*z)*(y*z)),\nu^L{}_A(y)\},\ \max\{\nu^U{}_A((x*z)*(y*z)),\nu^U{}_A(y)\}\},$$
$$\{\max\{\nu^L{}_B((x^1*z^1)*(y^1*z^1)),\nu^L{}_B(y^1)\},\max\{\nu^U{}_B((x^1*z^1)*(y^1*z^1)),\nu^U{}_B(y^1)\}\}$$

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$$=\{\max\{\max\{\nu^L{}_A((x*z)*(y*z)),\nu^L{}_B((x^1*z^1)*(y^1*z^1))\},\max\{\nu^L{}_A(y),\nu^L{}_B(y^1)\}\},$$
$$\max\{\max\{\nu^U{}_A((x*z)*(y*z)),\nu^U{}_B((x^1*z^1)*(y^1*z^1))\},\max\{\nu^U{}_A(y),\nu^U{}_B(y^1)\}\}\}$$
$$= r\ max\ \{(\bar{\nu}_A\times\bar{\nu}_B)(((x*z)*(y*z)),((x^1*z^1)*(y^1*z^1)),(\bar{\nu}_A\times\bar{\nu}_B)(y,y^1)\}$$

Hence A×B is an i-v intuitionistic fuzzy P-ideal of X× X

Definition 4.5:Let$\bar{\mu}_B,\bar{\upsilon}_B$ respectively, be an i-v membership and non membership function of each element xϵX to the set B.Then strongest i-v intuitionistic fuzzy set relationon X ,that is a membership function relation $\bar{\mu}_A$on$\bar{\mu}_B$ and non membership function relation $\bar{\upsilon}_A$ on$\bar{\upsilon}_B$ and μ_{A_B}, ν_{A_B} whose i-v membership and non membership function, of each element (x,y) ϵX×X and defined by $\bar{\mu}_{A_B}$ (x,y)=r min$\{\bar{\mu}_B(x),\bar{\mu}_B(y)\}$& $\bar{\nu}_{A_B}$ (x,y)=r max$\{\bar{\upsilon}_B(x),\bar{\upsilon}_B(y)\}$

Definition 4.6 Let B=[$\langle\mu_B^L,\mu_B^U\rangle$, $\langle\upsilon_B^L,\upsilon_B^U\rangle$] be an i-v subset in a set X, then the strongest i-v intuitionistic fuzzy relation on X that is a i-v A on B is A_B and defined by, $A_B=[\langle\mu^L_{A_B},\mu^U_{A_B}\rangle,\langle\nu^L_{A_B},\nu^U_{A_B}\rangle]$

Theorem 4.7 Let $B=[\langle\mu^L_{A_B},\mu^U_{A_B}\rangle,\langle\nu^L_{A_B},\nu^U_{A_B}\rangle]$be an i-v subset in a set X and $A_B=[\langle\mu^L_{A_B},\mu^U_{A_B}\rangle,\langle\nu^L_{A_B},\nu^U_{A_B}\rangle]$ be the strongest i-v intuitionistic fuzzy relation on X. then B is an i-v intuitionistic P-ideal of X if and only if A_B is an i-v intuitionistic fuzzy P-ideal of X×X.

Proof: Let B be an i-v intuitionistic fuzzy a-ideal of X. then$\bar{\mu}_{AB}(0,0)$=r min$\{\bar{\mu}_B(0),\bar{\mu}_B(0)\}$
≥r min$\{\bar{\mu}_B(x),\bar{\mu}_B(y)\}=\bar{\mu}_{AB}(x,y)$ and $\bar{\upsilon}_{AB}(0,0)$=r max$\{\bar{\upsilon}_B(0),\bar{\upsilon}_B(0)\}$≤r max$\{\bar{\upsilon}_B(x),\bar{\upsilon}_B(y)\}=\bar{\upsilon}_{AB}(x,y)$ ∀(x,y) ϵX×X.

On the other hand $\bar{\mu}_{A_B}$ (x_1,x_2)=r min $\{\bar{\mu}_B(x_1),\bar{\mu}_B(x_2)\}$

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≥r min{r min$\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\}$,r min$\{\bar{\mu}_B((x_2*z_2)*(y_2*z_2)),\bar{\mu}_B(y_2)\}$}
=r min{r min$\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B((x_2*z_2)*(y_2*z_2))\}$,r min $\{\bar{\mu}_B(y_1),\bar{\mu}_B(y_2)\}$}
=r min $\{\bar{\mu}_{AB}((x_1*z_1)*(y_1*z_1),(x_2*z_2)*(y_2*z_2)),\bar{\mu}_{AB}(y_1,y_2)\}$
=r min $\{\bar{\mu}_{AB}(((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_{1,}z_2))),\bar{\mu}_{AB}(y_1,y_2)\}$

Also, $\bar{\nu}_{A_B}$ (x_1,x_2)=r max $\{\bar{\upsilon}_B(x_1),\bar{\upsilon}_B(x_2)\}$
≤ r max{r max$\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\upsilon}_B(y_1)\}$,r max$\{\bar{\upsilon}_B((x_2*z_2)*(y_2*z_2)),\bar{\upsilon}_B(y_2)\}$}
= r max{r max$\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\upsilon}_B((x_2*z_2)*(y_2*z_2))\}$,r max $\{\bar{\upsilon}_B(z_1),\bar{\upsilon}_B(z_2)\}$}
=r max$\{\bar{\upsilon}_{AB}((x_1*z_1)*(y_1*z_1),(x_2*z_2)*(z_2*y_2)),\bar{\upsilon}_{AB}(y_1,y_2)\}$
=r max$\{\bar{\upsilon}_{AB}(((x_1,x_2)*(z_1,z_2))*((y_1,y_2)*(z_{1,}z_2))),\bar{\upsilon}_{AB}(z_1,z_2)\}$

For all $(x_1,x_2),(y_1,y_2),(z_1,z_2)$ in X×X. hence A_B is an i-v intuitionistic fuzzy P-ideal of X×X.
Conversely,
let A_B be an i-v intuitionistic fuzzy P-ideal of X×X. then for all (x,x)ϵX×X.we have
r min $\{\bar{\mu}_B(0),\bar{\mu}_B(0)\}=\bar{\mu}_{AB}(0,0)\geq\bar{\mu}_{AB}(x,x)$= r min$\{\bar{\mu}_B(x),\bar{\mu}_B(x)\}(or)\bar{\mu}_B(0)\geq\bar{\mu}_B(x)$ and
r max $\{\bar{\upsilon}_B(0),\bar{\upsilon}_B(0)\}=\bar{\upsilon}_{AB}(0,0)\leq\bar{\upsilon}_{AB}(x,x)$=rmin$\{\bar{\upsilon}_B(x),\bar{\mu}_B(x)\}(or)\bar{\upsilon}_B(0)\leq\bar{\upsilon}_B(x)$∀x$\epsilon$X. Now,
Let $(x_1,x_2),(y_1,y_2),(z_1,z_2)$ ϵX×X, then
rmin $\{\bar{\mu}_B(x_1,x_2)\}=\bar{\mu}_{AB}(x_1,x_2)\geq$r min $\{\bar{\mu}_{AB}(((x_1,x_2)*((z_1,z_2))*((y_1,y_2)*(z_1,z_2))),\bar{\mu}_{AB}(y_1,y_2)\}$
=r min $\{\bar{\mu}_{AB}((x_1*z_1)*(y_1*z_1),(x_2*z_2)*(y_2*z_2)),\bar{\mu}_{AB}(y_1,y_2)\}$
=r min {r min $\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\}$,r min $\{\bar{\mu}_{AB}((x_2*z_2)*(y_2*z_2)),\bar{\mu}_B(y_2)\}$}

Also, rmax $\{\bar{\upsilon}_B(x_1,x_2)\}=\bar{\upsilon}_{AB}(x_1,x_2)$
≤r max $\{\bar{\upsilon}_{AB}(((x_1,x_2)*((z_1,z_2))*((y_1,y_2)*(z_1,z_2))),\bar{\upsilon}_{AB}(y_1,y_2)\}$
=r max $\{\bar{\upsilon}_{AB}(((x_1*z_1)*(y_1*z_1)),((x_2*z_2)*(y_2*z_2))),\bar{\upsilon}_{AB}(y_1,y_2)\}$
=r max {r max $\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\}$,r max $\{\bar{\upsilon}_{AB}((x_2*z_2)*(y_2*z_2)),\bar{\upsilon}_B(y_2)\}$}

If $x_2=y_2=z_2=0$, then
r min $\{\bar{\mu}_B(x_1),\bar{\mu}_B(0)\}\geq$ r min {r min $\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\},\bar{\mu}_B(0)\}$ and
r max $\{\bar{\upsilon}_B(x_1),\bar{\upsilon}_B(0)\}\geq$ r max {r max $\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\upsilon}_B(y_1)\},\bar{\upsilon}_B(0)\}$
$\bar{\mu}_B(x_1)\geq$r min $\{\bar{\mu}_B((x_1*z_1)*(y_1*z_1)),\bar{\mu}_B(y_1)\}$ and
$\bar{\upsilon}_B(x_1)\geq$r max $\{\bar{\upsilon}_B((x_1*z_1)*(y_1*z_1)),\bar{\upsilon}_B(y_1)\}$.
Therefore B is i-v intuitionistic fuzzy P-ideal of X.

Theorem 4.8: If$\bar{\mu}_A$ is a i-v intuitionistic fuzzy a-ideal of BCI-algebra X, then $\bar{\mu}_{A^m}$ is also i-v intuitionistic fuzzy P-ideal of BCI-algebra X

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Proof: For all x, y, z $\in$ X

1. $\bar{\mu}_A(0) \geq \bar{\mu}_A(x),\ \bar{\nu}_A(0) \leq \bar{\nu}_A(x).\quad [\bar{\mu}_A(0)]^m \geq [\bar{\mu}_A(x)],\ [\bar{\nu}_A(0)]^m \leq [\bar{\nu}_A(x)]$

$\bar{\mu}_A(0)^m \geq \bar{\mu}_A(x)^m,\ \bar{\nu}_A(0)^m \leq \bar{\nu}_A(x)^m.\quad \bar{\mu}_{A^m}(0) \geq \bar{\mu}_{A^m}(x),\ \bar{\nu}_{A^m}(0) \leq \bar{\nu}_{A^m}(x)\ \ \forall x \in X$

2. $\bar{\mu}_A(x) \geq r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}.\ [\bar{\mu}_A(x)]^m \geq [r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}]^m$

$\bar{\mu}_A(x)^m \geq r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}^m.\ \bar{\mu}_{A^m}(x) \geq r\min\{\bar{\mu}_A((x*z)*(y*z))^m, \bar{\mu}_A(y)^m\}$

$\bar{\mu}_{A^m}(x) \geq r\min\{\bar{\mu}_{A^m}((x*z)*(y*z)), \bar{\mu}_{A^m}(y)\}$

3. $\bar{\nu}_A(x) \leq r\max\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}.\ [\bar{\nu}_A(x)]^m \leq [r\max\{\bar{\nu}_A((x*z)*(y*z))\bar{\nu}_A(y)\}]^m$

$\bar{\nu}_A(x)^m \leq r\max\{\bar{\nu}_A((x*z)*(y*z)), \bar{\nu}_A(y)\}^m.\ \bar{\nu}_{A^m}(x) \leq r\max\{\bar{\nu}_A((x*z)*(y*z))^m, \bar{\nu}_A(y)^m\}$

$\bar{\nu}_{A^m}(x) \leq r\max\{\bar{\nu}_{A^m}((x*z)*(y*z)), \bar{\nu}_{A^m}(y)\}$

Theorem 4.9: If $\bar{\mu}_A$ is a i-v intuitionistic fuzzy R-ideal of BCI-algebra X, then $\bar{\mu}_{A\cap B}$ isalso a i-v intuitionistic fuzzy P-ideal of BCI-algebra X

Proof: For all x, y, z $\in$ X

1. $\bar{\mu}_A(0) \geq \bar{\mu}_A(x),\ \bar{\nu}_A(0) \leq \bar{\nu}_A(x)\ \textit{and}\ \bar{\mu}_B(0) \geq \bar{\mu}_B(x),\ \bar{\nu}_B(0) \leq \bar{\nu}_B(x)$

$\min\{\bar{\mu}_A(0), \bar{\mu}_B(0)\} \geq \min\{\bar{\mu}_A(x), \bar{\mu}_B(x)\}, \min\{\bar{\nu}_A(0), \bar{\nu}_B(0)\} \leq \min\{\bar{\nu}_A(x), \bar{\nu}_B(x)\}$

$\bar{\mu}_{A\cap B}(0) \geq \bar{\mu}_{A\cap B}(x), \bar{\nu}_{A\cap B}(0) \leq \bar{\nu}_{A\cap B}(x)$

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2. $\bar{\mu}_A(x) \geq r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}, \bar{\mu}_B(x) \geq r\min\{\bar{\mu}_B((x*z)*(y*z)), \bar{\mu}_A(y)\}$

$\{\bar{\mu}_A(x), \bar{\mu}_B(x)\} \geq \{r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}, r\min\{\bar{\mu}_B((x*z)*(y*z)), \bar{\mu}_B(y)\}\}$

$\min\{\bar{\mu}_A(x), \bar{\mu}_B(x)\} \geq \min\{r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\}, r\min\{\bar{\mu}_B((x*z)*(y*z)), \bar{\mu}_B(y)\}\}$

$\geq \min\{r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_B((x*z)*(y*z))\}, r\min\{\bar{\mu}_A(y), \bar{\mu}_B(y)\}\}$

$\bar{\mu}_{A\cap B}(x) \geq r\min\{\bar{\mu}_{A\cap B}((x*z)*(y*z)), \bar{\mu}_{A\cap B}(y)\}$

3. $\bar{\nu}_A(x) \leq r\max\{\bar{\nu}_A((x*z)*(y*z)), \bar{\nu}_A(y)\}, \bar{\nu}_B(x) \leq r\max\{\bar{\nu}_B((x*z)*(y*z)), \bar{\nu}_A(y)\}$

$\{\bar{\nu}_A(x), \bar{\nu}_B(x)\} \leq \{r\max\{\bar{\nu}_A((x*z)*(y*z)), \bar{\nu}_A(y)\}, r\max\{\bar{\nu}_B((x*z)*(y*z)), \bar{\nu}_B(y)\}\}$

If one is contained in the other

$\min\{\bar{\nu}_A(x), \bar{\nu}_B(x)\} \leq \min\{r\max\{\bar{\nu}_A((x*z)*(y*z)), \bar{\nu}_A(y)\}, r\max\{\bar{\nu}_B((x*z)*(y*z)), \bar{\nu}_B(y)\}\}$

$\bar{\nu}_{A\cap B}(x) \leq r\max\{\min\{\bar{\nu}_A((x*z)*(y*z)), \bar{\nu}_B((x*z)*(y*z))\}, \min\{\bar{\nu}_A(y), \bar{\nu}_B(y)\}\}$

$\bar{\nu}_{A\cap B}(x) \leq r\max\{\bar{\nu}_{A\cap B}((x*z)*(y*z)), \bar{\nu}_{A\cap B}(y)\}$

Theorem 4.10: If $\bar{\mu}_A$ is a i-v intuitionistic fuzzy P-ideal of BCI-algebra X, then $\bar{\mu}_{A\cup B}$ isalso a i-v intuitionistic fuzzy P-ideal of BCI-algebra X.

Proof: For all x, y, z $\in$ X

1. $\bar{\mu}_A(0) \geq \bar{\mu}_A(x),\ \bar{\nu}_A(0) \leq \bar{\nu}_A(x)\ \textit{and}\ \bar{\mu}_B(0) \geq \bar{\mu}_B(x),\ \bar{\nu}_B(0) \leq \bar{\nu}_B(x)$

$\min\{\bar{\mu}_A(0), \bar{\mu}_B(0)\} \geq \min\{\bar{\mu}_A(x), \bar{\mu}_B(x)\},\ \min\{\bar{\nu}_A(0), \bar{\nu}_B(0)\} \leq \min\{\bar{\nu}_A(x), \bar{\nu}_B(x)\}$

$\bar{\mu}_{A\cup B}(0) \geq \bar{\mu}_{A\cup B}(x), \bar{\nu}_{A\cup B}(0) \leq \bar{\nu}_{A\cup B}(x)$

2. $\bar{\mu}_A(x) \geq r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\},\quad \bar{\mu}_B(x) \geq r\min\{\bar{\mu}_B((x*z)*(y*z)), \bar{\mu}_A(y)\}$

$\{\bar{\mu}_A(x), \bar{\mu}_B(x)\} \geq \{r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\},\ r\min\{\bar{\mu}_B((x*z)*(y*z)), \bar{\mu}_B(y)\}\}$

$\max\{\bar{\mu}_A(x), \bar{\mu}_B(x)\} \geq \max\{r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_A(y)\},\ r\min\{\bar{\mu}_B((x*z)*(y*z)), \bar{\mu}_B(y)\}\}$

$\geq \max\{r\min\{\bar{\mu}_A((x*z)*(y*z)), \bar{\mu}_B((x*z)*(y*z))\},\quad r\max\{\bar{\mu}_A(y), \bar{\mu}_B(y)\}\}$

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If one is contained in the other

$$\mathrm{r}\min\{\max\{\bar{\mu}_A((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\mu}_B((x*z)*(\mathrm{y}*\mathrm{z}))\}, \quad \max\{\bar{\mu}_A(\mathrm{y}), \bar{\mu}_B(\mathrm{y})\}\}$$

$$\bar{\mu}_{A\cup B}(x) \geq r\min\{\bar{\mu}_{A\cup B}((x*\mathrm{z})*(\mathrm{y}*\mathrm{z})), \bar{\mu}_{A\cup B}(\mathrm{y})\}$$

$$3. \bar{\nu}_A(x) \leq r\max\{\bar{\nu}_A((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\nu}_A(\mathrm{y})\}, \quad \bar{\nu}_B(x) \leq r\max\{\bar{\nu}_B((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\mu}_A(\mathrm{y})\}$$

$$\{\bar{\nu}_A(x), \bar{\nu}_B(x)\} \leq \{r\max\{\bar{\nu}_A((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\nu}_A(\mathrm{y})\}, r\max\{\bar{\nu}_B((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\nu}_B(z)\}\}$$

$$\max\{\bar{\nu}_A(x), \bar{\nu}_B(x)\} \leq \max\{r\max\{\bar{\nu}_A((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\nu}_A(\mathrm{y})\}, r\max\{\bar{\nu}_B((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\nu}_B(\mathrm{y})\}\}$$

$$\bar{\nu}_{A\cup B}(x) \leq r\max\{\max\{\bar{\nu}_A((x*z)*(\mathrm{y}*\mathrm{z})), \bar{\nu}_B((x*z)*(\mathrm{y}*\mathrm{z}))\}, \max\{\bar{\nu}_A(\mathrm{y}), \bar{\nu}_B(\mathrm{y})\}\}$$

$$\bar{\nu}_{A\cup B}(x) \leq r\max\{\bar{\nu}_{A\cup B}((x*\mathrm{z})*(\mathrm{y}*\mathrm{z})), \bar{\nu}_{A\cup B}(\mathrm{y})\}$$

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